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make $k' \cos \vartheta' = (1 + \lambda k \cos \vartheta)/(1 + \lambda)$, $k' \sin \vartheta' = \lambda k \sin \vartheta/(1 + \lambda)$, the equations of the third curve become

$$\begin{aligned} x_3 &= a' + k' \cos \vartheta' \cdot f(t) - k' \sin \vartheta' \cdot \varphi(t), \\ y_3 &= b' + k' \sin \vartheta' \cdot f(t) + k' \cos \vartheta' \cdot \varphi(t). \end{aligned} \tag{3}$$

These equations are of the form (2) and consequently represent a curve similar to (1) and (2).

437. Proposed by J. BROOKS SMITH, Hampden Sidney, Va.

Let D, E, F be three arbitrary points taken on the sides of a triangle ABC. If Δ and Δ' be the areas of the triangles ABC and DEF, show that

$$\frac{\Delta'}{\Delta} = \frac{AF \cdot BD \cdot CE + AE \cdot CD \cdot BF}{abc},$$

the sign of each factor being determined as follows: Each segment adjacent to one of the vertices of the triangle ABC is to be regarded as positive or negative according as it is drawn towards or from the other vertex of the side containing the segment.

SOLUTION BY PAUL CAPRON, Annapolis, Md.

Let $AF = m$, $AE = q$, $BF = r$, $BD = k$, $CD = p$, $CE = l$; and let the areas of the triangles be $AFE = \Delta_1$, $BDF = \Delta_2$, $CED = \Delta_3$. Then, a, b, c being positive we have, in accordance with the given convention of signs, and the further convention that the area of a Δ shall be positive if the cyclic arrangement of its vertices is contra-clockwise, negative otherwise:

$$\frac{\Delta_1}{\Delta} = \frac{mq}{bc}, \qquad \frac{\Delta_2}{\Delta} = \frac{kr}{ca}, \qquad \frac{\Delta_3}{\Delta} = \frac{lp}{ab};$$

$$\frac{\Delta - \Delta_1 - \Delta_2 - \Delta_3}{\Delta} = \frac{\Delta'}{\Delta} = \frac{abc - amq - bkr - clp}{abc}.$$

Substituting $a = k + p$, $b = l + q$, $c = m + r$, we have

$$\frac{\Delta'}{\Delta} = \frac{klm + pqr}{abc} = \frac{AF \cdot BD \cdot CE + AE \cdot CD \cdot BF}{abc}.$$

Also solved by S. W. REAVES, H. C. FEEMSTER, C. N. SCHMALL, HORACE OLSON, and the PROPOSER.

438. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

By means of the theorem that the product of the diagonals of a quadrilateral inscribed in a circle is equal to the sum of the two products of pairs of opposite sides, obtain the usual formulas for $\sin(\alpha \pm \beta)$ and $\cos(\alpha \pm \beta)$ in terms of $\sin \alpha$, $\sin \beta$, $\cos \alpha$, $\cos \beta$.

SOLUTION BY A. M. HARDING, University of Arkansas.

Suppose PR is a diameter and $\angle SPR = \alpha$ and $\angle RPQ = \beta$. (Fig. 1.) Then

$$PR \cdot SQ = PS \cdot RQ + PQ \cdot SR$$

or

$$2r \cdot 2r \sin(\alpha + \beta) = 2r \cos \alpha \cdot 2r \sin \beta + 2r \cos \beta \cdot 2r \sin \alpha.$$

Hence,

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta.$$

Suppose PQ is a diameter and $\angle SPQ=\alpha$ and $\angle RPQ=\beta$. (Fig. 2.) Then

$$PR\cdot QS=PS\cdot RQ+PQ\cdot SR$$

or

$$2r\cos\beta\cdot 2r\sin\alpha=2r\cos\alpha\cdot 2r\sin\beta+2r\cdot 2r\sin(\alpha-\beta).$$

Hence,

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta.$$

Suppose PQ is a diameter and $\angle RPQ=\alpha$ and $\angle SQP=\beta$. (Fig. 3.)

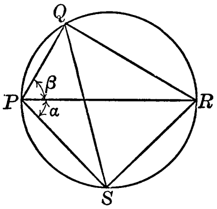

FIG. 1.

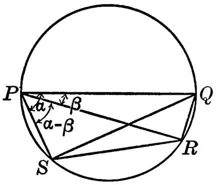

FIG. 2.

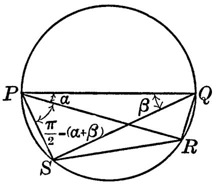

FIG. 3.

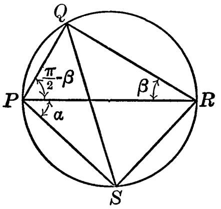

FIG. 4.

Then

$$PR\cdot SQ=PS\cdot RQ+PQ\cdot SR$$

or

$$2r\cos\alpha\cdot 2r\cos\beta=2r\sin\beta\cdot 2r\sin\alpha+2r\cdot 2r\sin[\pi/2-(\alpha+\beta)].$$

Hence,

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta.$$

Suppose PR is a diameter and $\angle SPR=\alpha$ and $\angle PRQ=\beta$. (Fig. 4.) Then

$$PR\cdot SQ=PS\cdot RQ+PQ\cdot SR$$

or

$$2r\cdot 2r\sin[\pi/2+(\alpha-\beta)]=2r\cos\alpha\cdot 2r\cos\beta+2r\sin\beta\cdot 2r\sin\alpha.$$

Hence,

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta.$$

Also solved by C. N. SCHMALL.

439. Proposed by CHARLES N. SCHMALL, New York City.

Show that the areas of any two triangles circumscribed about the same circle are in the same ratio as their perimeters.

SOLUTION BY A. L. MCCARTY, Cape Girardeau, Mo.

Let the radius of the circle be r and the sides of the circumscribed triangles be a, b, c and d, e, f respectively.

Now it is evident that the area of the first triangle is $\frac{1}{2}r(a+b+c)$ and the area of the second is $\frac{1}{2}r(d+e+f)$. Hence, the two triangles are to each other as their perimeters.